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Fourier Interpolation To The External Forces For Seismic Response Analyses And The Availabilities

M. Inukai

National Institute for Land and Infrastructure Management,
Ministry of Land, Infrastructure, Transport and Tourism, Japan

1. Introduction

In dynamic analyses, the linear acceleration methods are mostly used and these stabilities are required for the external forces, more than 300 gal.

Therefore, in this study, seismic response analyses were executed using the observed seismic accelerations.

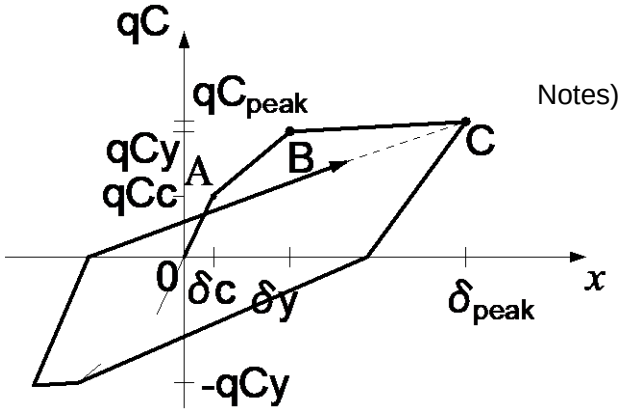
During these analyses, the external forces for the yield point, peak point, etc. between the sampling time were calculated by the Fourier Interpolation. Also, these availabilities were discussed.

2. Seismic Response Analyses

The equation of motion about SDOF and Hysteresis property of Tri-Linear.

$$m\ddot{x} + c\dot{x} + kx = -m\ddot{y}$$

m : mass(N/(cm/sec^2))
c : damping factor(N/(cm/sec))
k : stiffness(N/cm)
x : relative displacement(cm)
$\dot{x}$: relative velocity(cm/sec)
$\ddot{x}$: relative acceleration(cm/sec^2)
$\ddot{y}$: horizontal acceleration record on the ground surface (cm/sec^2)



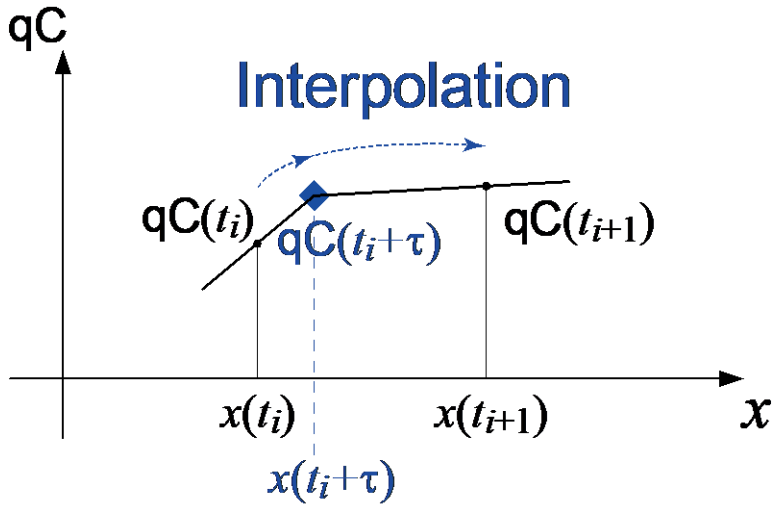
Notes) (1) Takeda Model
(2) Point A: Cracking point, B: Yield point, C: Peak.
(3) Shear Coefficient qC is the division of the storey shear force (Q) by the weight.
(4) Q : Storey Shear Force ($=k\cdot x$)

3. Fourier Series

3.1. Interpolation by Fourier Series

The analyzed time at the points of stiffness changing were not always same as the sampling time of the observed seismic accelerations.

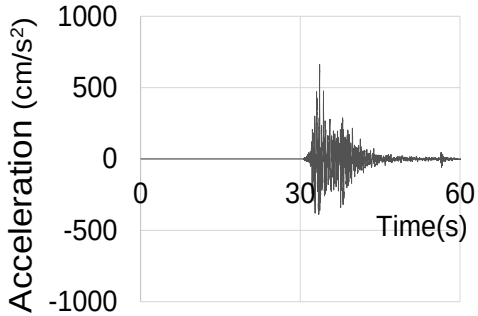
These interpolations were analyzed by Fourier Series.



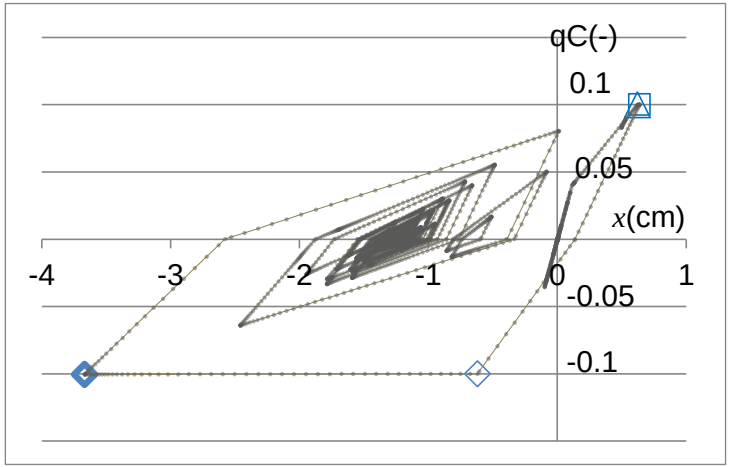
Interpolation (◆) near the point of stiffness changing.

3. Fourier Series

3.1. Interpolation by Fourier Series



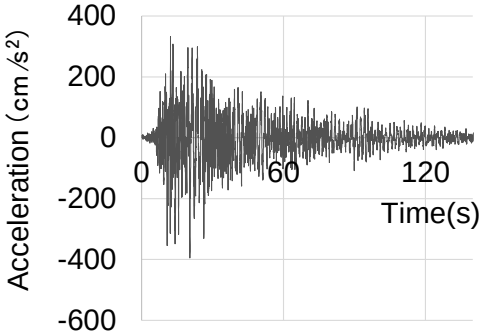
AQV EW, 2009 L'Aquila Earthquake, Italy

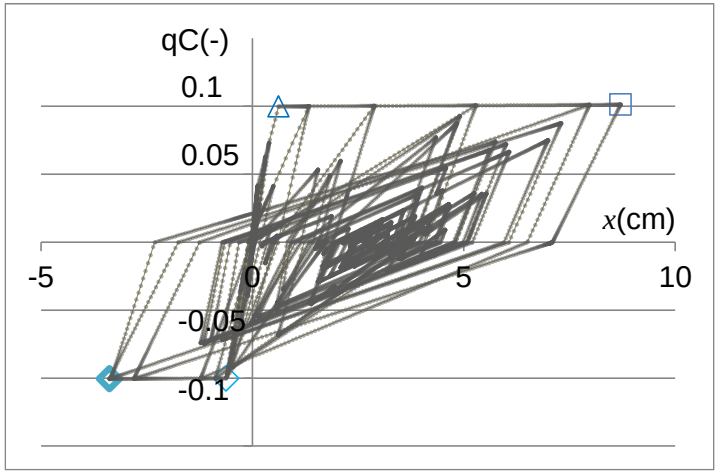


(T=0.5 sec, h=0.05, qCy=0.1)

3. Fourier Series

3.1. Interpolation by Fourier Series



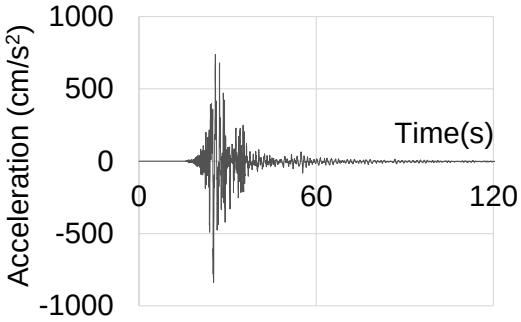


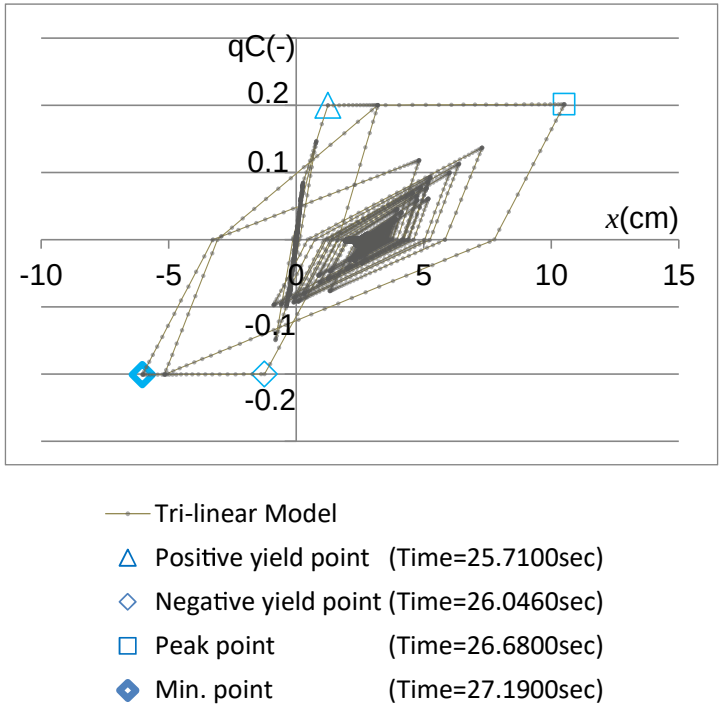
Concepción NS, 2010 Maule Earthquake, Concepción Centro, Chile

(T=0.5 sec, h=0.05, qCy=0.1)

3. Fourier Series

3.1. Interpolation by Fourier Series





Abira Hayakita EW, 2018 Hokkaido Eastern Iburi Earthquake, Hokkaido, Japan

(T=0.5 sec, h=0.05, qCy=0.2)

3. Fourier Series

3.2. Fourier Series and FFT

Observed seismic accelerations $\ddot{y}$ were expressed by the discrete digital values in both of Fourier Series and FFT.

$$\ddot{y} = \ddot{y}_1, \ddot{y}_2, \cdots, \ddot{y}_N \equiv \ddot{y}_t$$

⇒ Δt: data sampling time ($t = \beta\Delta t$) were divided by 20 like β = 0.00, 0.05, 0.10, ···, 0.90, 0.95, 1.00, 1.05, 1.10, ···, 1.90, 1.95, 2.00, 2.05, 2.10, 2.15, 2.20, ···, N -1.

Fourier Series

N :number of data (even number), Δt : data sampling time ($t = \beta\Delta t$, β = 0,1,2, ···, N -1, Integer), α = 0,1,2, ···, N/2, Integer.

$$A_\alpha = \frac{2}{N}\sum_{\beta=0}^{N-1} \ddot{y}_t \cos\left(2\pi\frac{\alpha\beta}{N}\right) \qquad B_\alpha = \frac{2}{N}\sum_{\beta=0}^{N-1} \ddot{y}_t \sin\left(2\pi\frac{\alpha\beta}{N}\right)$$

$$\ddot{y}_t = \frac{A_0}{2} + \sum_{\alpha=1}^{N/2-1}\left[A_\alpha \cos\left(2\pi\frac{\alpha\beta}{N}\right) + B_\alpha \sin\left(2\pi\frac{\alpha\beta}{N}\right)\right] + \frac{A_{N/2}}{2}\cos\left\{2\pi\frac{(N/2)\beta}{N}\right\}$$

FFT

N :number of data (even number), Δt : data sampling time ($t = \beta\Delta t$, α ,β = 0,1,2, ···, N -1, Integer), the imaginary unit (i = $\sqrt{-1}$).

$$C_\alpha = \frac{1}{N}\sum_{\beta=0}^{N-1}\left[\ddot{y}_t \, \mathrm{e}^{-\mathrm{i}\left(2\pi\frac{\alpha\beta}{N}\right)}\right] \qquad \ddot{y}_t = \sum_{\alpha=0}^{N-1}\left[C_\alpha \, \mathrm{e}^{+\mathrm{i}\left(2\pi\frac{\alpha\beta}{N}\right)}\right]$$

Relationship between coefficients

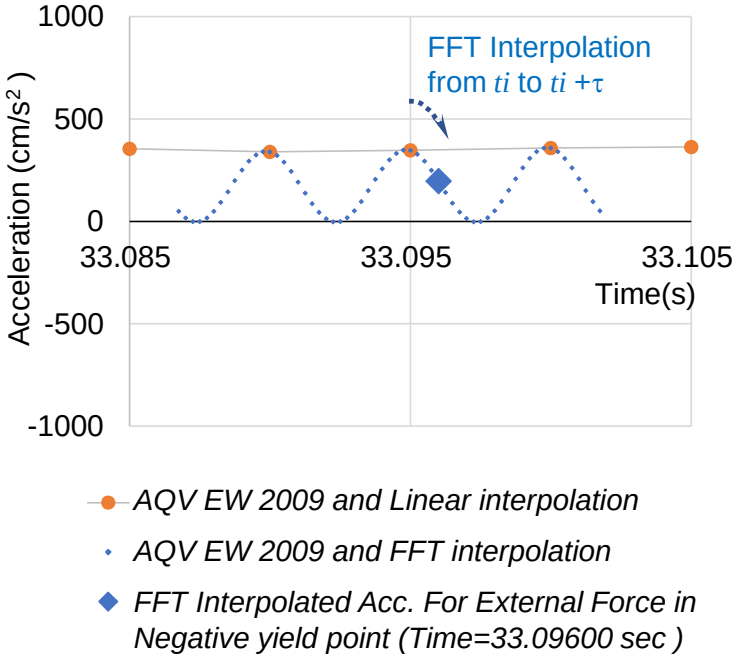
$$C_\alpha = \frac{A_\alpha - \mathrm{i}B_\alpha}{2}$$

3. Fourier Series

3.2. Fourier Series and FFT

FFT interpolated points have some band around the observed accelerations, not near the linear interpolation lines.

The Figure shows the FFT Interpolated Acceleration was less than the observed data and the linear interpolation.



AQV EW, 2009 L'Aquila Earthquake, Italy

4. Conclusions

(1) Analyses of SDOF were executed using some of the maximum accelerations observed in 2009, 2010 and 2018.

(2) The hysteresis property was Tri-linear Model. At the points where the stiffness changed, the external forces were analysed by the FFT interpolations.

(3) At the negative yield points and many points where the stiffness changed, FFT interpolations were applied and these interpolate values were available.

(4) The availability of the application of FFT interpolations were discussed appropriately.

Thanks