Appendix 12

Reference materials of JRC-IPSC

# A DISPLACEMENT-BASED MDOF TECHNIQUE TO ACCOUNT FOR THE EFFECTS OF INFILLS IN FRAMED STRUCTURES

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**ABSTRACT**: A response spectrum procedure for the global analysis of multi-degree-offreedom building structures to account for the nonlinear behaviour of structural members as a function of increasing earthquake intensities is presented. The procedure is based on interstorey secant stiffness and damping envelopes and searches for a displacement shape compatible with the structure's secant stiffness and spectral response for the considered earthquake intensity. The procedure is used to study the behaviour of regular and irregular infilled RC building structures in the context of performance based design.

Key Words: performance based design, infill walls, multi-degree-of-freedom response spectrum analysis

## INTRODUCTION

The effect of nonstructural masonry infills on the global dynamic behaviour of framed structures is still a controversial issue (Colombo et al. 1998). Even though infills are disregarded in the design process by most codes, they are capable of modifying the behaviour of building structures to a large extent.

The detrimental effects of irregularly arranged infill panels are known, and attempts to account for these effects in design have been made (Fardis et al. 1999a). Even though the relative importance of plan-wise and height-wise irregularities is not completely clear (some studies seem to indicate that the effects of irregularities in plan are not as severe as those of irregularities in elevation) (Fardis et al. 1999b), there is a general consensus about the need to take into account the effects of irregular distributions of infills in design.

On the other hand, the need to account for regularly arranged infills is not very evident. Indeed, the effect of regular infill patterns is typically regarded as positive. Infills can make the structure considerably stiffer and stronger (which is a positive effect in most cases), and can significantly contribute to energy dissipation by progressive damage of the panels, thus protecting the frame from larger damage. The consequence of this point of view is that there is no need to account for the effects of infills in design (as long as they are regularly arranged), and that the infills can be regarded as a second line of defence, which may eventually improve the global seismic behaviour of the structure.

Pseudodynamic tests conducted on a four-storey reinforced concrete (RC) frame have thrown more light into this problem (Negro and Colombo 1997). The seismic response corresponded to a storey-wise progressive failure of the panels, thus transforming the structure into a soft-storey mechanism. Even though the characteristics of the input did not result in excessive deformations, this indicated that regular infill patterns can produce an irregular response. A confirmation of this finding came from the analysis of the damage resulting from the Koçaeli (1999) Earthquake (Dolsek and Fajfar 2001). As an effect of such earthquake, many apparently well designed and constructed

uniformly-infilled frames suffered extensive damage, often leading to the collapse of the first storey, and analyses indicated that the reason for the collapse may have remained within the infill panels, in spite of their regular arrangement (see Photo 1, showing a building in Gölçüc which pancaked during the 1999 Koçaeli earthquake with a sort of soft first storey effect, in spite of the fact it was uniformly infilled).



Photo 1 RC Building in Gölçüc which pancaked during the 1999 Koçaeli earthquake.

The storey-wise progression of the failure of the panels cannot be traced by standard singledegree-of-freedom (SDOF) techniques. A new assessment method, based on a simplified multidegree-of-freedom (MDOF) displacement-based technique, is used to study the behaviour of a threestorey RC infilled frame which will be subjected to pseudodynamic tests. The conditions which correspond to a storey-level mechanism are analysed, and the consequences for the structural behaviour of the frame are discussed. The technique is proposed as a viable means to account for these effects in analysis and design.

# ANALYSIS METHODOLOGY

The assessment of the response of a building structure can be performed by means of analytical procedures that can vary in complexity as a function of the methodology used and the level of refinement desired for the computed structural response.

Non-linear dynamic time history analysis by means of Finite Element Models (FEM) offers to date the most realistic description of the response of a structure to earthquake excitation. However, the limitations of such a methodology are many: the constitutive relations that represent the physical properties of the elements that make up the structure can be very complex and not always consider all the factors that determine their behaviour; the analysis procedure is computationally expensive, requires specialised and experienced engineers and may not always lead to a solution; finally the confidence of the available data used as input in the analysis is generally lower than the accuracy of the computed response. For these reasons parametric analyses are prohibitively expensive, thus excluding the possibility of using FEM non-linear dynamic analysis for design purposes.

Response Spectrum Analysis (RSA) methodologies offer a good compromise between accuracy and computational cost. The maximum response is obtained based on fundamental properties of the structure and of the seismic input. This makes the procedure ideal for design, as it allows to perform parametric analyses at different levels of seismic input and different structural configurations.

The limit of RSA lies in the approximations involved in determining the two quantities that govern the response of the structure: period of vibration and equivalent damping. In traditional forcebased design methods the period of the structure is obtained either by empirical expressions that take into account the geometry of the structure or by computing the elastic first mode of vibration. A constant level of equivalent damping is assigned as a function of the materials used, while the effect of energy dissipation due to the development of plastic behaviour is taken into account by reducing the elastic spectral forces. Whereas this approach may give a good description of the response of a regular structure with well distributed damage, it falls short in identifying the members that most contribute to energy dissipation, the mode of failure of irregular structures and the actual displacements obtained for the different members of the structure.

The methodology proposed herein is based on response spectrum analysis applied to MDOF systems and on equivalent secant stiffness and damping of the structure as a function of displacement response (Taucer 2000, Taucer et al. 2000). The methodology identifies the effective contributions to damping and stiffness of the different elements that form the structure, thus tracing the damage evolution and failure modes as a function of the earthquake intensity. The proposed approach is ideal for the analysis of infilled regular/irregular structures for which the stiffness and damping contributions of the infill wall can be explicitly taken into account.

## Non-linear Response Spectrum Analysis of SDOF Systems

It has been well recognised that the earthquake response of a non-linear SDOF system can adequately be well represented by means of linear analysis of an equivalent system with secant stiffness and hysteretic damping obtained at maximum response (Miranda and Ruiz-Garcia, 2002). Whereas some differences are obtained in the time history response of the non-linear and equivalent linear systems, a very good match is obtained for maximum response, in general sufficient for preliminary assessment and design. It follows that response spectrum analysis of the equivalent linear system will give a good approximation of the maximum earthquake response of the non-linear system.

Let us take a SDOF system described by an inertial mass m and by a given cyclic nonlinear forcedisplacement constitutive law. Furthermore, the system is discretised into force-displacement and damping-displacement envelope functions  $f_V$  and  $f_D$ . The force envelope is computed as the resisting force F developed by the system at increasing levels of displacement, considering either monotonic or cyclic behaviour of the constitutive law at the displacement of interest; similarly, the damping envelope is obtained by computing the hysteretic damping with the following expression:

$$\xi_h = \frac{W_D}{4\pi W_s} \tag{1}$$

where  $W_D$  is the energy contained by the hysteresis loop of the constitutive law and  $W_s$  is the elastic strain energy stored in the system at the considered displacement amplitude d.

The step-by-step procedure to determine the response of a nonlinear system to earthquake excitation is as follows:

Step 1.	Assume trial displacement $d^*$
Step 2.	$F = f_{V}[d^{*}]$ and $\xi_{eq} = f_{D}[d^{*}]$

Step 3.
$$k_{eq} = \frac{F}{d^*}$$
Step 4. $T_{eq} = 2\pi \sqrt{\frac{m}{k_{eq}}}$ Step 5. $S_d = RS_d [a_g, T_{eq}, \xi_{eq}]$ Step 6.If  $\frac{|S_d - d^*|}{S_d} < Tol$  Then $d = S_d$ , analysis has converged.else $d^* = S_d$ , go to Step 2

The procedure is iterative; when convergence is achieved with tolerance *Tol*, the resulting displacement corresponds to the response of the system to a seismic input of peak ground acceleration  $a_g$ . Spectral displacement  $S_d$  in *Step 5* is computed from displacement spectra  $RS_d$  corresponding to a specific soil class and to other parameters that may describe the ground motion.

The step-by-step procedure is repeated for different levels of  $a_g$  to obtain the response of the system with increasing levels of the earthquake intensity. Moreover, it is also possible to compute the total damping as the sum of hysteretic and viscous damping, thus giving a better control of the variables that determine the response of the system.

For a SDOF system made up of resisting elements in parallel the procedure is straight forward: there are as many force and damping envelope functions as there are elements in parallel and the total force is obtained as the sum of the resisting forces computed at displacement *d*. The equivalent damping of the system is computed as the ratio between the weighted sum of the damping contributions of each element in terms of their stored energy and the total elastic energy stored in the system.

For a system formed by  $M_q$  members ranging from q = 1 to Q, the total equivalent damping is computed as (Priestley and Calvi 1997):

$$\xi_{eq} = \frac{\sum_{q=1}^{Q} F_{M_q} d \xi_{M_q}}{\sum_{q=1}^{Q} F_{M_q} d}$$
(2)

The possibility of assembling the contributions of different elements in parallel is well suited for the analysis of infilled frames, offering the possibility to distinguish the contributions of the infill walls and of the RC frame.

#### Non-linear Response Spectrum Analysis of MDOF Systems

The transition from non-linear spectral analysis of SDOF systems to MDOF systems is not a trivial one. A MDOF system can be seen as a system in series, where the displacement shape is not known in advance. For the case of multi-storey buildings it is possible to discretise the structure as a system in series made up of as many elements as the number of storeys of the building, and as a system in parallel for the different elements that make up each storey (i.e., infill walls and RC frame elements are subjected to the same interstorey displacement).

Having presented in the previous sub-section the approach to compute the SDOF response of system in parallel, the next step is to compute the MDOF seismic response corresponding to a system in series. In fact, as acknowledged by many researchers, this is one of the main problems that has been faced by displacement based methodologies: the definition of the displacement shape (Faifar and Krawinkler 1997). A traditional option has been to assume a near to inverted triangular shape for regular buildings, to concentrate most of the deformation where a soft-storey mechanism is expected, or to assume a displaced shape based on capacity design considerations (Miranda 1997, Fajfar et al. 1997, Fardis and Panagiotakos 1997, Priestley and Calvi 1997, Reinhorn 1997). However, these assumptions use as premise the expected response of the structure, which is what the analysis methodology is expected to compute. As an alternative to overcome this problem the following iterative procedure is proposed:

Step 1.	Assume a trial displacement shape.
Step 2.	Compute the resisting force and equivalent damping of all members of the structure by means of the force and damping envelopes as a function of the trial displacement shape.
Step 3.	Assemble the element stiffness into the structure stiffness matrix.
Step 4.	Compute the equivalent damping of the structure by means of Eq. (2).
Step 5.	Compute the eigenvalues and eigenvectors of the structure based on the stiffness matrix computed in <i>Step 3</i> .
Step 6.	Enter the response spectra for a given earthquake intensity and compute the modal spectral displacements as a function of the modal periods computed in <i>Step 5</i> assuming constant damping (as computed in <i>Step 4</i> ) for all modes of vibration of the structure.
Step 7.	Compute the modal displacements of the structure based on the eigenvectors computed in <i>Step 5</i> and the spectral displacements computed in <i>Step 6</i> ; obtain the displaced shape by SRSS combination (for faster convergence it is also possible to account for the contribution of the first mode only).
Step 8.	Compare the obtained displaced shape with the trial shape. If the comparison is within the desired tolerance the solution converges, otherwise update the trial displaced shape with the computed displaced shape in <i>Step 7</i> and go to <i>Step 2</i> .

As with the SDOF system, an assumption is made for the displaced shape, which is updated through the iterative procedure until a solution is found. A set of iterations is performed for each level of the earthquake intensity, using as starting trial displaced shape the converged displaced shape at the previous earthquake intensity.

In essence, the procedure consists in searching a displacement shape that results in a stiffness matrix and equivalent damping such that, when computing the modal properties and entering the response spectra for a given earthquake intensity, a displacement response equal to the trial displacement shape is obtained.

#### Step-by-Step Analysis Procedure for Multi-Storey Building Structures

The procedure presented for MDOF systems is now presented for the particular case of multistorey structures. The first assumption that is made is that the behaviour of the building structure can be discretised as shear type, where independent interstorey shear-displacement envelope functions can be computed for each storey.

One way of establishing these envelope functions is by performing pushover analyses on a nonlinear model of the building, where unit interstorey displacements are imposed at the storeys of interest. Another way of calculating these functions is by computing the maximum capacity of the frame at each storey as a function of the member cross sections, setting yield interstorey displacements as a function of interstorey height and interstorey drifts at yield, and establishing the slope of the plastic branch as a function of the detailing or type of cross section of members.

The condensation of the response of a shear type building in the lateral degrees-of-freedom (DOF's) is exact when working in the linear range; however, when working in the nonlinear range the secant stiffness matrix obtained from the pushover analysis or from the stiffness-strength-ductility evaluations changes as a function of the displacement shape considered in the stiffness evaluation itself. However, these changes are usually not too large (Miranda and Ruiz-Garcia, 2002), therefore at this stage of analysis the assumption of being able to calculate nonlinear force-displacement envelopes from unit interstorey deformations is considered satisfactory.

The procedure for computing the response of a multistorey building follows the same step-by- step schemes presented in the previous section. The following nomenclature is introduced, where index i denotes the storey number (or mode number) and n the total number of storeys (or total number of modes considered in the analysis):

•	Storey displacement vector:	Ψ
•	Interstorey displacement vector:	Δψ
•	Member interstorey shear force envelopes:	$V_{Mq} = f_{VMq} [\Delta \psi]$
•	Member equivalent damping envelopes:	$\boldsymbol{\xi}_{\boldsymbol{M}\boldsymbol{q}} = f_{\boldsymbol{\xi}\boldsymbol{M}\boldsymbol{q}}\left[\boldsymbol{\Delta}\boldsymbol{\psi}\right]$

All vectors are of *n* (number of storeys) dimension. The building is composed of *Q* members  $M_q$  acting in parallel at each storey level. For example, a reinforced concrete (RC) infilled frame would be composed by two  $M_q$  members (Q = 2), namely the RC frame ( $M_1$ ) and the infill walls ( $M_2$ ). For storey levels where not all  $M_q$  members are present, zero interstorey force and damping functions are assigned.

Secant stiffness  $\mathbf{K}_{sec}$  is computed by assembling through the *n* storeys of the structure interstorey secant stiffness  $k_{sec}$  *i*, obtained as the sum of secant stiffness  $k_{sec}$   $M_{q}$  *i* calculated for each of the *Q* members working in parallel at storey level *i*. The secant stiffness of each member  $M_q$  is calculated as the ratio between resisting force  $V_{Mq}$  *i* and interstorey displacement  $\Delta \psi_i$ . The structure stiffness matrix  $\mathbf{K}_{sec}$  is then assembled as (for ease of notation, the term  $k_{sec}$  *i* in Eq. (3) was replaced by  $k_i$ ):

$$\mathbf{K}_{\text{sec}} = \begin{bmatrix} k_1 + k_2 & -k_2 & & & \\ -k_2 & & & & \\ & & -k_i & & \\ & & -k_i & k_i + k_{i+1} & -k_{i+1} & & \\ & & & -k_{i+1} & & \\ & & & & -k_n & \\ & & & & -k_n & k_n \end{bmatrix}$$
(3)

The mass matrix **M** is assumed diagonal, with all cross terms equal to zero and diagonal terms  $m_{ii}$  equal to storey mass  $m_i$ .

The input data given by the user to start up the analysis procedure are:

•	Number of storeys:	п
•	Member interstorey shear force envelopes:	$f_{VMq}$
•	Member equivalent damping envelopes:	$f_{\xi Mq}$
•	Storey masses:	$m_i$
•	Response spectrum function:	$S_a = RS_a [a_g, T, \xi]$
•	Set of target $a_g$ (of length K):	$a_g$
•	Trial displacement shape:	$\mathbf{\Psi}_0$
•	Convergence tolerance:	Tol
•	Structure viscous damping:	ξv

The step-by-step procedure is described as follows:

Step 1.
 Set 
$$k = 1$$

 Step 2.
  $a_g^k = a_{g_k}$ ;  $j = 1$ 

*Step 3.* Set trial displacement shape:

If 
$$k = 1$$
 and  $j = 1$  then  
 $(\Psi^{j})^{k} = \Psi_{0}$   
If  $k \neq 1$  and  $j = 1$  then  
 $(\Psi^{j})^{k} = \Psi^{k-1}$   
If  $k \neq 1$  and  $j \neq 1$  then  
 $(\Psi^{j})^{k} = (\Psi^{j-1})^{k}$ 

Step 4. Compute interstorey displacements  $(\Delta \psi^i)^k$  corresponding to  $(\psi^i)^k$ :

 $(\Delta \psi_1^{j})^k = (\psi_1^{j})^k$ ;  $(\Delta \psi_i^{j})^k = (\psi_i^{j})^k - (\psi_{i-1}^{j})^k$  for i = 2 to n

Step 5. Evaluate member interstorey forces  $(V_{Mq i})^k$  at each storey *i*:

$$(V_{Mq\,i}^{j})^{k} = f_{VMq} [(\Delta \psi_{i}^{j})^{k}]$$

*Step 6.* Compute member secant stiffness  $(k_{sec Mq i})^{j}$  at each storey *i*:

$$(k_{\sec Mq\,i}^{\ \ j})^{k} = \frac{(V_{Mq\,i}^{\ \ j})^{k}}{(\Delta \psi_{i}^{\ \ j})^{k}}$$

Step 7. Compute interstorey secant stiffness  $(k_{sec i})^{j}$  at each storey *i*:

$$(k_{\text{sec }i}^{j})^{k} = \sum_{Mq}^{Q} (k_{\text{sec }Mq}^{j})^{k}$$

*Step 8.* Assemble member secant stiffness  $(k_{sec i})^{jk}$  to obtain structure stiffness  $(\mathbf{K}_{sec})^{jk}$ .

*Step 9.* Evaluate equivalent damping  $(\xi_{Mq_i})^k$  at each storey *i*:

$$(\xi_{Mq\,i})^{k} = f_{\xi\,Mq} \left[ \left( \Delta \psi_{i}^{J} \right)^{k} \right]$$

*Step 10.* Compute the equivalent damping  $(\xi^{j})^{k}$  of the structure:

$$(\xi^{j})^{k} = \frac{\sum_{i=Mq}^{n} \sum_{Mq}^{Q} (\xi_{Mq\,i}^{j})^{k} [(V_{Mq\,i}^{j})^{k} (\Delta \psi_{Mq}^{j})^{k}]}{\sum_{i=Mq}^{n} \sum_{Mq}^{Q} (V_{Mq\,i}^{j})^{k} (\Delta \psi_{Mq}^{j})^{k}} + \xi_{v}$$

Step 11. Solve the eigenvalue problem  $\| (\mathbf{K}_{sec}^{\ j})^k - [(\omega_i^{\ j})^k]^2 \mathbf{M} \| = \mathbf{0}$  and obtain:

Modal angular frequencies  $(\omega_i^{j})^k$  and Modal shapes  $(\mathbf{\varphi}_i^{j})^k$ 

Step 12. Compute modal spectral accelerations  $(S_{a_i}^{j})^k$  corresponding to  $(T_i^j)^k$  and  $(\xi^j)^k$  as a function of  $a_g^k$ :

$$(T_i^{j})^k = \frac{2\pi}{(\omega_i^{j})^k} \quad ; \quad (S_{ai}^{j})^k = RS_a[a_g^{k}, (T_i^{j})^k, (\xi^{j})^k]$$

*Step 13.* Compute modal spectral displacement  $(S_{di}^{j})^k$ :

$$(S_{di}^{j})^{k} = \frac{(S_{ai}^{j})^{k}}{[(\omega_{i}^{j})^{k}]^{2}}$$

Step 14. Compute structure modal displacements  $(\Psi_i^j)^k$  corresponding to  $(S_d_i^j)^k$ :

$$(Y_{i}^{j})^{k} = \frac{(L_{i}^{j})^{k}}{(M_{i}^{j})^{k}} \quad ; \quad (L_{i}^{j})^{k} = (\varphi_{i}^{j})^{kT} \mathbf{M} \{\mathbf{l}\} \quad ; \quad (M_{i}^{j})^{k} = (\varphi_{i}^{j})^{kT} \mathbf{M} (\varphi_{i}^{j})^{k}$$
$$(\Psi_{i}^{j})^{k} = (\varphi_{i}^{j})^{k} (Y_{i}^{j})^{k} (S_{di}^{j})^{k}$$

Step 15. Compute the displacements  $(\psi^{*j})^k$  of the structure by SRSS combination of  $(\psi_i^{j})^k$ :

$$(\mathbf{\psi}^{*j})^k = SRSS\left[(\mathbf{\psi}_i^{j})^k\right]$$

Step 16. Compare computed displacements  $(\boldsymbol{\psi}^{*j})^k$  with trial displacements  $(\boldsymbol{\psi}^j)^k$ :

If 
$$\frac{(\Psi_i^{\ j})^k - (\Psi_i^{\ *j})^k}{(\Psi_i^{\ j})^k} \le Tol \quad \text{for all } i \text{ DOF}$$
  
go to  $Step 3$ 

else

$$({\bf \psi}^{j})^{k} = ({\bf \psi}^{*j})^{k}$$
, go to *Step 17*

Step 17. Compute external storey forces  $\mathbf{F}_{ext}^{k}$ :

$$\mathbf{F}_{ext}^{k} = \mathbf{K}_{sec}^{k} \mathbf{\psi}^{k}$$

Step 18. Compute modal effective mass  $M_{effi}^{k}$ :

$$M_{eff\,i}^{\ \ k} = \frac{(L_i^{\ \ k})^2}{M_i^{\ \ k}}$$

Step 19. Compute modal base shear  $V_{bi}^{k}$ :

$$V_{bi}^{\ k} = M_{effi} S_{ai}^{\ k}$$

Step 20. Compute total base shear  $V_b^k$  by SRSS combination:

$$V_{h}^{k} = SRSS \left[ V_{hi}^{k} \right]$$

*Step 21.* 

If 
$$k < K$$
 then

$$k = k + 1$$
, go to Step 2

else

Stop

The step-by-step procedure consists of an internal iteration loop denoted by index j and an external cycle loop denoted by index k. The external cycle loop consists of K cycles, each cycle corresponding to a target level of base acceleration  $a_g$ . At each cycle k,  $J^k$  iterations are performed to reach a converged solution.

From *Step 3* the trial displacement shape is taken either as the last converged state in the previous k cycle or as the last computed state obtained in the previous j iteration. As for the trial displacement shape  $\Psi_0$  used to start the procedure it is preferable to use an inverted triangular shape with small displacement values corresponding to elastic behaviour of the structure.

From *Step 10* the total damping of the structure is computed as the sum of the energy-weighted hysteretic damping contributions of structural members and the structure viscous damping  $\xi_{\nu}$  that remains constant throughout the analysis.

In *Step 13* the spectral displacements are computed from the spectral accelerations computed on *Step 12* from the acceleration spectra  $RS_a$ . This option enables the user to use the acceleration spectra as defined by most seismic building codes.

The variables introduced in *Step 14*, namely  $L_i$ ,  $M_i$  and  $Y_i$ , are no more than the modal-earthquake excitation factor, modal mass and modal amplitude used in the standard analysis of earthquake response of lumped MDOF systems.

It is also possible to compute other quantities of interest, such as the total interstorey force, or the percentage of equivalent damping proportioned by each storey or by each member type  $M_q$ .

# **EXAMPLE OF A 3-STOREY INFILLED RC BUILDING FRAME**

An example of a three storey frame RC building (Photo 2) that will be tested at the European Laboratory for Structural Assessment (ELSA) is presented in the following. The structure is part of a

project to study the seismic behaviour of flat-slab buildings designed in accordance with the 1986 Italian national seismic code (Ministero dei Lavori Pubblici 1986). Extensive nonlinear analyses have been performed in preparation for the testing campaign, thus permitting to derive the force and damping envelopes required for the proposed procedure (Negro et al. 2002). In addition, the analytical results will be published before performing the tests in the laboratory, thus giving the opportunity of a "blind" check of the validity of the assessment procedure proposed herein.



Photo 2 3-Storey RC Flat-slab Building

# **Description of the structure**

The test specimen is a full scale building composed of two frames with two spans of 6 and 4 meters as shown in Fig. 1. The storey heights are 2.82, 5.76 and 8.70 metres measured from the base of columns, with free interstorey heights of 2.70 m. A slab with a thickness of 20 cm and with 4 cm topping was adopted. The beams are 1 m wide, have the same height of the slab and are supported by columns of 40 cm square cross section. An eccentricity of 20 cm exists between the axis of the beam and that of the column. Due to the limited cross section height, beams have rather high reinforcement on both sides, however, only some rebars are anchored or passing through the column the column joint.

The self weight of the slab is  $3.5 \text{ kN/m}^2$ . An extra permanent load of  $2.0 \text{ kN/m}^2$  and a live load of  $2.0 \text{ kN/m}^2$  were considered. The inertial masses *m* used for the seismic design and analysis were of 51.61 Ton for the first and second storeys and of 54.12 Ton for the third storey. The structure was designed for medium seismicity (base shear coefficient 0.07, importance factor 1.0), which corresponds to a peak ground acceleration of 0.25 g.



Fig. 1 Lay-out of the RC building frame mock-up

To represent the construction practice before the new Italian code came into effect (Ministero dei Lavori Pubblici 1997), the detailing rules in the current code were intentionally violated. This applies to the eccentricity between beam and column axes, as well as to the width of the beam, which would not have been acceptable. In addition, no rules for ductility were considered: columns have single 8 mm stirrups (with 90° bents) at 20 cm spacing, beams have double 8 mm stirrups at 15 cm spacing. Standard materials were used (concrete C25/30 and steel deformed bars with 440 MPa characteristic yield strength).

## **Seismic Input**

The seismic input used for the analysis corresponds to the elastic response spectrum  $RS_a$  given by (Eurocode 8 1998) for sub-soil class B for increasing levels of peak ground acceleration  $a_g$ . The damping correction factor  $\eta$  is given by Eq. (4) and was derived as the best fit of the reduction factors proposed by (Boomer and Elnashai 1999) for elastic displacement spectra predicated from attenuation equations. Eq. (4) gives a better estimate than Eurocode 8 of the damping correction factor for large values of damping up to 30%.

$$\eta = \sqrt{\frac{7}{2+\xi}} \qquad \text{for} \quad \xi < 5\%$$

$$\eta = \sqrt{\frac{10}{5+\xi}} \ge 0.53 \quad \text{for} \quad \xi \ge 5\%$$
(4)

## **Interstorey RC Frame Envelopes**

Interstorey envelopes were derived for the RC frame based on analytical results obtained from a nonlinear model of the structure using the FEM computer code IDARC2D (Valles et al. 1996). The standard lumped-plasticity model was used using a trilinear model with pinching behaviour and strength and stiffness degradation; the skeleton curve was modified to include the effect of slippage of

the rebars. The parameters of the model were adjusted to fit the experimental behaviour observed on similar RC elements (Negro et al. 2002).

Nonlinear quasi-static cyclic analyses were performed by imposing displacement shapes corresponding to (d, d, d), (0, d, d) and (0, 0, d) for the first, second and third storeys to study the interstorey force-displacement behaviour; displacement *d* corresponded to a cyclic history of increasing amplitudes of 2.0, 5.0, 9.7, 14.8, 20.9 (18.0 for the third interstorey) and 27.0 mm. Three cycles were imposed at each displacement amplitude with the purpose of stabilising the pinching, stiffness and strength degradation effects accounted by the model; the interstorey force-displacement envelope used in this study was obtained from the values obtained at the third cycle.

## Force-Displacement Envelope

The interstorey shear-displacement envelopes obtained from the nonlinear analysis were fitted with the expression formulated by (Menegotto and Pinto 1973) to describe the monotonic envelope of the stress-strain uniaxial behaviour of steel reinforcing bars:

$$V_{c} = k_{c0} \left[ b_{V} + \frac{1 - b_{V}}{\left[ 1 + \left( \frac{\Delta \Psi}{d_{Vc0}} \right)^{R_{V}} \right]^{1/R_{V}}} \right] \Delta \Psi > 0$$
(5)



Fig. 2 RC Frame Interstorey Shear-Displacement Envelope

where  $V_c$  is the RC frame resisting force corresponding to interstorey displacement  $\Delta \psi$ ,  $k_{c0}$  is the initial stiffness,  $b_V$  is the post-elastic to initial stiffness ratio,  $d_{Vc0}$  is the "yield" interstorey displacement and  $R_V$  is a parameter which can vary from 0 to infinity. Low values of the  $R_V$  parameter result in a smooth variation of the slope from initial to post-elastic stiffness, while large values of  $R_V$  give a sharp variation of the slope resulting in a curve that mimics a bi-linear behaviour. The advantage of this formulation is that it is continuous and closed-form and requires parameters that are well related with

the force-displacement envelope of a structure. Eq. (5) corresponds to the member interstorey shear force envelope function  $f_{VMq}$  proposed in the step-by-step procedure. For ease of notation index  $M_q$  has been changed to *c* to denote the RC frame. The force-displacement envelopes are shown in Fig. 2 and the parameters used in Eq. (5) are given in Table 1.

Storey	<b>k</b> <sub>co</sub> kN/mm	b <sub>v</sub>	<b>d</b> <sub>V c0</sub> mm	Rv
1	113.4	-0.038	7.0	1.6
2	69.35	0.054	6.2	4.0
3	42.35	-0.086	8.5	5.0

 Table 1
 RC Frame Menegotto-Pinto Parameters of Interstorey Shear-Displacement Envelopes

# Damping-Displacement Envelope

The interstorey damping-displacement envelope is computed from Eq. (1) based on the area contained by the hysteresis loops of the cyclic numerical nonlinear response of the numerical model described in the previous sub-section using IDARC2D. The equivalent damping was computed for the third cycle at each of the imposed displacement amplitudes, resulting in lower values than those expected for the first cycle, thus recognising some amount of degradation in the RC frame. The damping envelopes were also fitted with the Menegotto and Pinto formulation, the expression is now reformulated into Eq. (6) to account for the different parameters given as input to define the member storey shear force envelope function  $f_{\xi Mq}$  proposed in the step-by-step procedure:

$$\xi_{c} = \frac{\xi_{c0}}{d_{\xi c0} - d_{\xi cs}} \left[ b_{\xi} + \frac{1 - b_{\xi}}{\left[ 1 + \left( \frac{\Delta \psi - d_{\xi cs}}{d_{\xi c0} - d_{\xi cs}} \right)^{R_{\xi}} \right]^{1/R_{\xi}}} \right] (\Delta \psi - d_{\xi cs}) > 0 \quad \text{for} \quad \Delta \psi \ge d_{\xi cs} \tag{6}$$

$$b_{\xi} = \frac{\frac{\xi_{cu}}{d_{\xi cu} - d_{\xi cs}} - 1}{\frac{d_{\xi cu} - d_{\xi cs}}{d_{\xi c0} - d_{\xi cs}} - 1} \tag{7}$$

Table 2	RC Frame	Menegotto-Pinto	Parameters	of Interstorey	Damping-l	Displacement	Envelope
		0			1 0	1	1

Storey	d <sub>ξ cs</sub>	<b>d</b> <sub>ξ с0</sub> тт	d <sub>ξ cu</sub>	<b>5</b> c0	<b>5</b> cu	R <sub>ξ</sub>
1	2	7.2	27	8.4	5.9	5.0
2	2	6.0	27	5.0	5.8	3.5
3	2	4.8	8.7	8.5	3.2	2.5

where  $\xi_c$  is the RC frame equivalent damping corresponding to interstorey displacement  $\Delta \psi$ ,  $\xi_{c0}$  is the damping related to displacement  $d_{\xi c0}$  and  $R_{\xi}$  is a parameter which can vary from 0 to infinity.

Parameter  $b_{\xi}$  is computed from Eq. (7),  $\xi_{cu}$  is the damping related to displacement  $d_{\xi cu}$  and  $d_{\xi cs}$  is the displacement corresponding to loss of linearity; for  $\Delta \psi < d_{\xi cs}$  the hysteretic equivalent damping  $\xi_c$  is equal to zero. The damping-displacement envelopes are shown in Fig. 3 and the parameters used in Eqs. (6) and (7) are given in Table 2.



Fig. 3 RC Frame Interstorey Damping-Displacement Envelope

# **Interstorey Infill Wall Envelopes**

The infill wall envelopes were computed from the infill macromodel developed at the University of Patras by Panagiotakos and Fardis and described in (ECOEST-PREC8 1996).

## Force-Displacement Envelope

The force displacement envelope is a trilinear function with the following properties:

$$k_{w0} = \frac{G_w A_w}{H_w} \quad ; \quad A_w = L_w t_w \tag{8}$$

$$k_{wu} = \frac{E_w W_{eff} t_w}{\sqrt{L^2 + H^2}} \cos^2 \theta \quad ; \quad \theta = \arctan(H/L)$$
(9)

$$W_{eff} = 0.175 (\lambda_h H)^{-0.4} \sqrt{L^2 + H^2} \quad ; \quad \lambda_h = \sqrt[4]{\frac{E_w t_w \sin 2\theta}{4 E_c I_c H_w}}$$
(10)

$$V_{w0} = \tau_{wcr} A_w$$
 and  $V_{wu} = 1.3 V_{w0}$  (11)

where  $k_{w0}$  is the initial stiffness,  $k_{wu}$  is the secant ultimate stiffness at ultimate shear strength  $V_{wu}$  and  $V_{w0}$  is the shear cracking strength corresponding to the change of secant stiffness from initial to ultimate. The input parameters needed to compute these values are:  $G_w$ ,  $E_w$  and  $\tau_{wcr}$  corresponding to

the shear modulus (1240 MPa), elastic modulus in the horizontal (weak) direction (2520 MPa) and diagonal cracking strength (0.28 MPa) of masonry;  $L_w$ ,  $H_w$  and  $t_w$  corresponding to the length (3.60 and 5.60 m), height (2.70 m for all storeys) and thickness (11.2 cm) of the infill wall; H and L corresponding interstorey height (2.94 m for all storeys) and bay length (4 and 6 m) of the RC frame;  $E_c$  and  $I_c$  corresponding to the elastic modulus of concrete (30000 MPa) and the moment of inertia of columns (213000 cm<sup>4</sup>). Other parameters derived from the above equations are the cross section area  $A_w$  of the infill wall and the effective width  $W_{eff}$  of the equivalent strut inclined at an angle  $\theta$  with respect to the horizontal.

The linear unloading branch after reaching  $V_{wu}$  is replaced by the exponential strength decay proposed by (Klingner and Bertero 1976), where v is the strength decay coefficient (0.035 mm<sup>-1</sup>), v is the elongation of the compression strut and  $d_{wu}$  is the displacement corresponding to maximum strength  $V_{wu}$ :

$$V_{w} = V_{wu} e^{-vv} \quad \text{for} \quad \Delta \psi > d_{wu} \quad ; \quad v = \frac{\Delta \psi - d_{wu}}{\cos \theta} \quad \text{and} \quad d_{wu} = \frac{V_{wu}}{k_{wu}} \tag{12}$$

The shear-displacement envelope of the infill wall is shown in Fig. 4 and was computed as the sum in parallel of the contributions of the two sections of 3.60 and 5.60 meters of length. The parameters that result from Eqs. (9), (11) and (12) are given in Table 3 and describe the infill wall shear-displacement envelope  $f_{Vw}$  function used in the step-by-step procedure.



Fig. 4 Infill Wall Interstorey Shear-Displacement Envelope

 Table 3 Infill Wall Parameters of Interstorey Shear-Displacement Envelopes

Storey	<b>k</b> <sub>w0</sub>	k <sub>wu</sub>	V <sub>w0</sub>	V <sub>wu</sub>	θ	v
	kN/	/mm	k	N	rad	mm <sup>-1</sup>
1, 2, 3	473	52.4	289	375	0.53	0.035

It is also possible to reduce the shear capacity of the infill wall to account for cyclic damage by using the following expression, where *a* (~0.025) and  $\kappa$  (exponent that accounts for half cycle ductility accumulation) are parameters derived from tests and set to zero for the present analysis:

$$V_{w} = V_{w} e^{-2a(\Delta \psi/d_{w0})^{\kappa}} \quad ; \quad d_{w0} = \frac{V_{w0}}{k_{w0}}$$
(13)

#### **Damping Displacement-Envelope**

The damping envelope of the infill wall is computed from the expression given in Eq. (14) in terms of ductilities  $\mu_w$  and  $\mu_{wu}$  as defined in Eq. (15), stiffness ratios *p* and *p*<sub>1</sub> as defined in Eq. (16) and parameters  $\alpha$ ,  $\beta$  and  $\gamma$ , defining the unloading brach; the damping formulas apply for reloading cycles after the first. Damping  $\xi_w$  is equal to zero for  $\mu_w < 1$ .

The infill wall damping envelope function  $f_{\xi w}$  defined by Eq. (14) is shown in Fig. 5 and was derived with the parameters given on Table 4. The values of  $\alpha$ ,  $\beta$  and  $\gamma$  correspond to those recommended in (ECOEST-PREC8 1996).

$$\xi_{w} = \frac{(1-p)(\mu_{w}-1)}{\pi \mu} \frac{\beta + 0.5(1-\alpha)(1-\gamma)[1+p(\mu_{w}-1)]}{1+p(\mu-1)} \quad \text{for} \quad \mu_{w} < \mu_{wu}, \quad \text{else}$$

$$\xi_{w} = \frac{\mu_{w}-1-p(\mu_{wu}-1)+p_{1}(\mu_{w}-\mu_{wu})}{\pi \mu} \frac{\beta + 0.5(1-\alpha)(1-\gamma)[1+p(\mu_{wu}-1)-p_{1}(\mu_{w}-\mu_{wu})]}{1+p(\mu_{wu}-1)-p_{1}(\mu_{w}-\mu_{wu})}$$
(14)

$$\mu_{w} = \frac{\Delta \Psi}{d_{w0}} \quad \text{and} \quad \mu_{wu} = \frac{d_{wu}}{d_{w0}}$$
(15)

$$p = \frac{V_{wu} - V_{w0}}{d_{wu} - d_{w0}} \frac{1}{k_{wo}} \quad \text{and} \quad p_1 = \frac{V_{wu} \left(e^{-1.5 v d_{wu} / \cos \theta} - 1\right)}{1.5 d_{wu} k_{wo}}$$
(16)



Fig. 5 Infill Wall Interstorey Damping-Displacement Envelope

Table 4 Infill Wall Parameters of Interstorey Damping-Displacement Envelope



#### **Response of the Regularly and Irregularly Infilled RC Frame**

The proposed step-by-step procedure is used to analyse the response of the RC building considered in this example for a series of increasing peak ground accelerations  $a_g$  up to a maximum of 0.35g (length of  $a_g$  vector: K = 60, i.e. intervals of 0.00583g). The properties of the RC frame and infill walls were given in the previous sections. The remaining values given as input to start-up the analysis procedure are: number of storeys n = 3, trial displacement shape  $\psi_0$  equal to a constant drift of 0.05%, convergence tolerance *Tol* of 0.01% and constant viscous damping  $\xi_v$  of 2.5%.

The analysis results are presented in terms of interstorey displacements versus peak ground acceleration. The plots are presented against a performance criteria (a dashed line) for visually assessing the state of the structure, with no influence on the analysis results. The performance criteria corresponds to a polynomial curve defined by interstorey drifts of 0.15% (4.41 mm), 0.45% (13.3 mm) and 0.75% (22.1 mm) at  $a_g$  values of 0.07g, 0.25g and 0.35g, allowing for minor, medium and extensive damage for small, medium and large periods of return of the earthquake. For  $a_g$  less than 0.07g the interstorey drift criteria is constant and equal to 0.15%.

The results for the regularly infilled frame are shown in Fig. 6, and show that the structure satisfies the assumed performance criteria up to a maximum interstorey displacement at first storey of 8.2 mm, when the infill wall enters into the unloading branch and the structure develops an unstable soft-storey mechanism at 0.28g.

The analysis of the response of the bare frame and of irregular storey-wise infilled wall configurations give further insight into the problem. In Fig. 7 the response of the bare frame is shown, showing that interstorey drifts are largest at the second storey, exceeding the performance criteria at  $a_g$  equal to 0.175g with an interstorey displacement of 8.8 mm; the structure becomes unstable when the first storey also develops a mechanism at  $a_g$  equal to 0.25g. In Fig. 8 shows the response of the frame with infills at second and third storeys only, showing that in this case all deformations are concentrated in the first storey, exceeding the performance criteria at  $a_g$  equal to 0.16g with a displacement of 7.8 mm and becoming unstable at  $a_g$  equals to 0.19g.

On Table 5 the response quantities relative to the external storey force ( $F_{ext}$ ), interstorey shear ( $V_{int}$ ), interstorey secant stiffness ( $k_{sec int}$ ) and interstorey equivalent damping ( $\xi_{int}$ ) contributions of the RC frame and infill wall members for  $a_g$  equal to 0.175g are given for the three structural configurations corresponding to the regularly infilled frame, bare frame and first soft storey frame. The response quantities in terms of interstorey displacement, structure damping, first mode period, spectral displacement and spectral acceleration, and structure base shear are given in Table 6 for  $a_g$  equal to 0.175g for all the possible combinations of regularly and irregularly infilled frames.

Focusing on the first three configurations represented in Figs. 6, 7 and 8, it is possible to conclude what has been observed from previous experimental tests, nonlinear time history analyses and field observations: the regularly infilled frame can sustain base accelerations 50% larger than the bare frame, however, the failure mode of the latter is more "ductile" when compared with the sudden failure that takes place when the first storey infill wall reaches its maximum capacity. The soft storey configuration develops a mechanism at about the same base acceleration as the bare frame, yet with mechanisms that are intrinsically different: the bare frame develops a mechanism at the second storey, confirming the results obtained from preliminary time history nonlinear analyses.



Fig. 6 Regularly Infilled RC Frame Displacement Response



Fig. 7 Bare RC Frame Displacement Response



Fig. 8 1<sup>st</sup> Soft Storey RC Frame Displacement Response

Storey n	F <sub>ext</sub> (kN)		V <sub>int</sub> (kN)		k <sub>sec int</sub> (kN/mm)			ξ <sub>int</sub> (%)				
	Total	RC	Infill	Total	RC	Infill	Total	RC	Infill	Total	RC	Infill
Regularly Infilled												
3 <sup>rd</sup>	262	22	241	262	22	241	515	42	473	0.00	0.00	0.00
2 <sup>nd</sup>	228	155	74	491	176	315	192	69	123	1.11	0.09	1.01
1 <sup>st</sup>	127	120	7	618	296	322	199	95	104	1.81	0.50	1.31
Bare Frame												
3 <sup>rd</sup>	248	248	0	248	248	0	41	41	0	1.17	1.17	0.00
2 <sup>nd</sup>	170	170	0	418	418	0	48	48	0	2.06	2.06	0.00
1 <sup>st</sup>	82	82	0	500	500	0	70	70	0	2.94	2.94	0.00
1 <sup>st</sup> Soft Sto	orey											
3 <sup>rd</sup>	203	17	187	203	17	187	515	42	473	0.00	0.00	0.00
2 <sup>nd</sup>	187	75	112	390	92	298	295	69	225	0.21	0.00	0.21
1 <sup>st</sup>	163	462	-298	554	554	0	60	60	0	7.01	7.01	0.00

Table 5 RC Frame Response for 3 Infilled Configurations for  $a_g = 0.175$ g

	3 <sup>rd</sup>	yes	no	yes	yes	no	no	no	yes
Infill wall?	2 <sup>nd</sup>	yes	no	yes	no	yes	no	yes	no
	1 <sup>st</sup>	yes	no	no	yes	yes	yes	no	no
Interstorey Dis	slaceme	nt (mm)							
3 <sup>rd</sup> Storey		0.51	6.07	0.39	0.44	5.69	6.28	6.17	0.43
2 <sup>nd</sup> Storey		2.56	8.77	1.32	10.42	0.55	7.43	1.35	9.73
1 <sup>st</sup> Storey		3.11	7.14	9.19	1.66	0.71	1.24	7.72	7.99
ξ <sub>s</sub> (%)		5.42	8.66	9.72	7.06	8.12	7.46	9.33	8.74
1 <sup>st</sup> Mode Perio	od (s)	0.224	0.435	0.339	0.343	0.248	0.353	0.360	0.418
1 <sup>st</sup> Mode S <sub>d</sub> (n	nm)	5.36	17.58	10.27	11.67	5.83	12.16	11.74	16.17
1 <sup>st</sup> Mode S <sub>a</sub> (n	n/s²)	4.20	3.67	3.54	3.91	3.75	3.85	3.59	3.66
Base Shear (k	N)	618	500	554	477	369	430	517	525

Table 6 RC Frame Response for all Possible in-height Infilled Configurations /  $a_g = 0.175$ g

# CONCLUSIONS

The analysis methodology presented herein offers a valuable tool to analyse and assess the seismic response of multi-storey structures. The methodology is based on stiffness and equivalent damping envelopes in terms of the interstorey displacements of the different members that constitute the structure (RC and masonry walls for the example presented). The analysis procedure allows to study the evolution of structural response for increasing levels of earthquake base excitation to different structural configurations at very low computing costs, thus offering a valuable tool not only for assessment, but for structural design as well.

The results confirm the observations gathered in recent years from the study of infilled frames: the design of a RC frame must take into account the presence of infilled masonry walls in order to account for the effective modes of failure that take place at different levels of earthquake excitation. The proposed methodology offers the means to analyse/design such structures in the framework of performance base design.

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